Suppose that we have focused on location and species

Suppose that we have 3 groups and that, after we remove the i-th individual, we have .

Furthermore, after removing the i-th individual, we also have and

Here are the expressions after integrating out :

Multinomials are in red, Poissons are in black.

Let’s deal with the multinomial first, assuming we just have two groups to avoid this getting too big:

Start cutting pieces:

Use factorial and gamma function rules to simplify:

This suggests the following expression for the integrated multinomial piece:

Let’s deal with the Poisson part:

Let’s start cutting some pieces:

This suggests the following expression for the Poisson piece:

Combining these two pieces, we get: